COMMON CORE STATE STANDARDS FOR MATHEMATICS

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- Standards for Mathematical Practice
- Webb Leveling

  http://pirll.tie.wikispaces.net/file/view/7_WebbLevelDocuments.pdf
- Three Shifts in Mathematics
- Assessment—Smarter Balanced
  - DRAFT of Content Specifications for Summative Assessment of CCSS-M, http://www.smarterbalanced.org/wordpress/wp-content/uploads/2011/12/MathContentSpecifications.pdf
- Discussion

Standards for Mathematical Practice

Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.

Webb Leveling: Expectations for Student Performance

ACQUIRE		USE		EXTEND	
LEVEL 1: Recall	**LEVEL 2: Skill/Concept**		**LEVEL 3: Strategic Thinking**		**LEVEL 4: Extended Thinking**
Recall of a fact, information or procedure	Use information or conceptual knowledge, two or more steps, etc.		Requires reasoning, developing a plan or sequence of steps, some complexity, more than one possible answer		Requires an investigation, time to think and process multiple conditions of the problem
☐ Memorize ☐ Recall ☐ Perform Procedures ☐ Conduct Investigations ☐ Demonstrate/Explain	☐ Perform Procedures ☐ Conduct Investigations ☐ Demonstrate/Explain ☐ Demonstrate Understanding ☐ Communicate Understanding ☐ Analyze/Investigate		☐ Demonstrate Understanding ☐ Communicate Understanding ☐ Analyze/Investigate ☐ Conjecture ☐ Generalize ☐ Prove ☐ Analyze Information ☐ Evaluate		☐ Conjecture ☐ Generalize ☐ Prove ☐ Analyze Information ☐ Evaluate ☐ Solve ☐ Non-routine/make connections ☐ Apply concepts/make connections, ☐ Generate/create

NOTE: Although verbiage may indicate a lesson is written at a higher cognitive level, one must also consider the rigor (cognitive demand) and engagement expected of students. Examples:

Example 1: Students asked to create a list during a lesson would be demonstrating understanding at a Level 1, not a Level 4 as the verb *create* would indicate. A lesson written at a Level 4 would ask the students to create an original artifact that demonstrates higher order thinking skills.

Example 2: Asking students to solve a problem would be a Level 2 sample of communicating understanding. Having students solve a problem, explain the sequence of steps and prove their solution would be a Level 3 sample of communicating understanding.

Refer to the Descriptors and Questions for Webb Leveling guide for further details.

Adapted from Webb, Norman L. "Alignment, Depth of Knowledge, and Change," Wisconsin Center for Education Research, Florida Educational Research Association 50th Annual Meeting, 2005.

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THREE SHIFTS IN MATHEMATICS

- Focus strongly where the standards focus
- Coherence: Think across grades and link to major topics within grades
- Rigor: In major topics, pursue with equal intensity:
  - Conceptual understanding
  - Procedural skill and fluency
  - Application

Taken from Lessons Learned, a Smarter Balanced Webconference

Shift One: Focus strongly where the CCSS focus

- The CCSSM significantly narrow the scope of content and deepen how time and energy are spent in mathematics.
- Focus deeply only on what is emphasized in the standards, so that students gain strong foundations
- Two levels of focus:
  - What's in/What's out
  - The shape of the content that is in (Major/Supporting/Additional Targets)

Taken from Lessons Learned, a Smarter Balanced Webconference

Shift Two: Coherence
Think across grades, and link to major topics within grades

- Carefully connect the mathematics within and across grades so that students can build new understanding onto foundations built in previous years.
- Each year, we expect solid conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.

Taken from Lessons Learned, a Smarter Balanced Webconference

Shift Three: Rigor
Equal intensity in conceptual understanding, procedural skill/fluency, and application

- The CCSSM require a balance of:
  - Solid conceptual understanding
  - Procedural skill and fluency
  - Application of skills in problem solving situations
- This requires equal intensity in time, activities, and resources in pursuit of all three.

Taken from Lessons Learned, a Smarter Balanced Webconference

SOLID CONCEPTUAL UNDERSTANDING

- Standards require that students know more than "how to get the answer"
- Instructional and assessment tasks must provide access to concepts from a number of perspectives to show deep understanding
- Math is more than a set of mnemonics or discrete procedures
- Conceptual understanding supports other aspects of rigor (fluency & application)

SOLID CONCEPTUAL UNDERSTANDING

Questions assessing understanding of a mathematical concept can vary in length and format:

- Can be briefly stated and briefly answered questions that are simple to answer if a student understands the concept, and difficult to answer if the student does not—or slightly longer tasks that require students to explain.
- Long tasks probably drift away from the goal of assessing conceptual understanding of a particular mathematical concept as called for in the standards.

SOLID CONCEPTUAL UNDERSTANDING

Some possible rules of thumb for conceptual understanding:

- Ratchet down the computational skill required
- Have a concept in mind when you begin writing a task
- Understand the concept yourself. What are its basic components?
- Consider the 'efficiency' of the task – how many points are being generated in how much time? How important is the topic?
- Task could be very easy to solve if concept is understood by the student
- Important that the format of these tasks are non-routine (tasks that assess conceptual understanding are good candidate for technology enhancement for this reason)
- Assessing conceptual understanding is about understanding if a student can do pure mathematics. Does not include "transfer" to application setting.

FLUENCY

The standards require *speed* and *accuracy* in calculation.

Fluency is called for explicitly in certain standards.

Standard	Required Fluency
2.OA.2 2.NBT.5	Add/subtract within 20 (recall single-digit sums) Add/subtract within 100
3.OA.7 3.NBT.2	Multiply/divide within 100 (recall single-digit products) Add/subtract within 100
5.NBT.4	Add/Subtract within 1,000,000
5.NBT.5	Multi-digit multiplication
6.NS.2, 3	Multi-digit division Multi-digit decimal operations

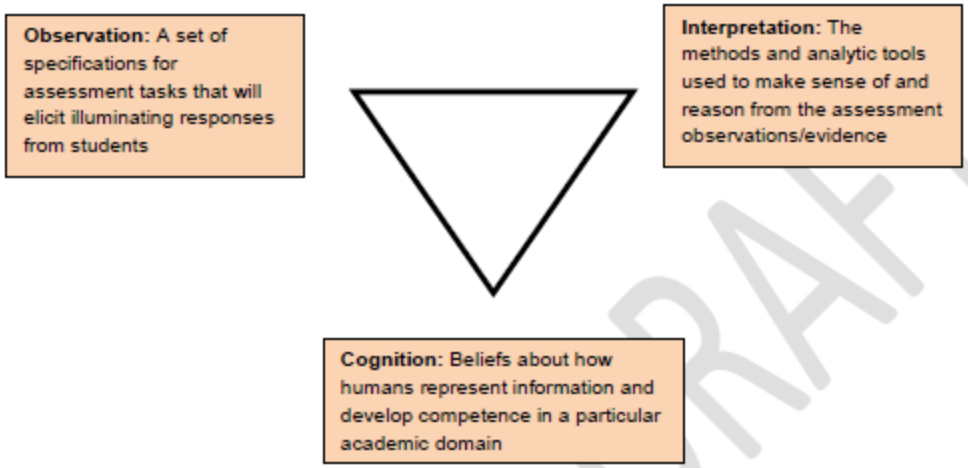
APPLICATION

- The standards require that students use appropriate concepts and procedures for application even when not promoted to do so.
- Provide opportunities at all grade levels for students to apply math concepts in “real world” situations, recognizing this means different things in K-5, 6-8, and HS.

Smarter Balanced Assessment Consortium—group that has contracted to create the assessments for CCSS-M for some states, including South Dakota, Montana, and Missouri.

Chris is on the Test Design work group.

Sharon is on the Performance Tasks work group.



The Assessment Triangle (NRC, 2001, p. 44)

Evidence-based design: Smarter Balanced is committed to using evidence-based design in its development of assessments in the Consortium's system. The Smarter Balanced approach is detailed in the following section, but a brief explanation is as follows. In this document, four "claims" are set forth regarding what students should know and be able to do in the domain of mathematics. Each claim is accompanied by a "Rationale" that provides the basis for establishing the claim as central to mathematics. The claims and Rationales represent the "cognition" part of the assessment triangle. For each claim and Rationale there is a section representing the "observation" corner of the triangle. Here, a narrative description lays out the kinds of evidence that would be sufficient to support the claim, which is followed by tables with "Assessment Targets" linked to the Common Core standards. Finally, the "interpretation" corner of the triangle is represented by a section for each claim that lists the "Proposed Reporting Categories" that the assessment would provide.

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Claims for Mathematics Summative Assessment

Overall Claim for Grades 3-8	"Students can demonstrate progress toward college and career readiness in mathematics."
Overall Claim for Grade 11	"Students can demonstrate college and career readiness in mathematics."
Claim #1	**Concepts & Procedures** "Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."
Claim #2	**Problem Solving** "Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."
Claim #3	**Communicating Reasoning** "Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Claim #4	**Modeling and Data Analysis** "Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

Claim 1: Concepts and Procedures	**Grade:** 8 – HS	**Comparison to Traditional State Test Item(s)**
Item Stem The equation of a circle on the coordinate plane is $(x-h)^2+(y-k)^2=r^2$ where h and k are the x and y coordinates of the center of the **circle** and r is its radius. A circle with radius 4 is centered at the origin. Give the y-values for all points on the circle with x-coordinate 1.		This item is similar to a traditional item, which is important to note. Not all items will be "new" or "innovative." They *will* provide evidence related to the Smarter Balanced claims and targets. Adaptive testing allows an item that would typically be given only on a high school test to be delivered in Grade 8. This item would not appear below Grade 8 because opportunity to learn (equations with exponents) plays a role in determining grade appropriateness of content. This is a constructed response item as opposed to a traditional selected response, which makes guessing a poor strategy.
Adaptive Versions Can also be given without the formula to make trigonometric approaches more likely.		The adaptive version (without the formula) differs from traditional items because it makes multiple approaches more feasible, particularly since the equation for a circle is not one that students typically memorize
Source: Smarter Balanced Assessment Consortium Supplemental Samples		

Claim 2: Problem Solving	**Grade:** HS	**Comparison to Traditional State Test Item(s)**
Item Stem A circle has center at (6, 7) and goes through the point (1, 4). Give the coordinates for the center of a circle that is tangent to the first at the point (1, 4) and half the area. Show your work or explain how you got your answer.		This item integrates content from across grades, a feature not very prevalent in existing assessment items, which are often designed with specific grade level focus. Students' understanding of each topic below contributes to his/her success in solving this problem: • Area of circle (Grade 7) • All circles are similar (HS) • Using similar triangles to understand slope (Grade 8) • Equation of a circle (HS) • Definition of tangent (HS)
Adaptive Versions Lower difficulty by replacing the "half the area" with "same area."		
Source: Smarter Balanced Assessment Consortium Supplemental Samples		

Claim 3: Communicating Reasoning	**Grades:** 7 – HS	**Comparison to Traditional State Test Item(s)**
Item Stem Isabel says that there is exactly one circle through any two distinct points. If Isabel is correct, explain why, using drawings as necessary. If Isabel is not correct, draw and label an example to show why she is not correct.		Traditional items have often been written to align neatly to individual standards or subparts of standards. This sample item captures the idea of understanding the circle (which students have been studying since Kindergarten, including an in depth study of angle measure through the lens of the circular arc in Grade 4). In Grade 7, the task could be aligned to the cluster level, "Draw, construct, and describe geometrical figures and describe the relationships between them," as well as one or more Standards for Mathematical Practice.
Adaptive Versions Change argument to say "three distinct points" OR change argument so that one of the two distinct points is the circle's center.		
Source: Smarter Balanced Assessment Consortium Supplemental Samples		

Claim 4: Modeling and Data Analysis	**Grades:** 8 – HS	**Comparison to Traditional State Test Item(s)**
Item Stem The picture shows children holding hands around the circumference of Earth. How many children holding hands would actually be needed to surround Earth? Write down all assumptions you make and mathematics you use to support your answer.  The volume of the earth is approximately 108.321 x 10^{10} km^3.		Items designed to collect evidence for Claim 4 represent a significant change in thinking. Many of these items/tasks ask students to make reasoned estimates or assumptions as part of the problem solving process. In this particular item, the student has to consider the "size" of a typical "child." The student is asked to write down his/her assumptions so that item scoring can factor in student assumptions. Claim 2 and Claim 4 items both require students to develop chains of autonomous reasoning, but Claim 2 items are considered well-posed, whereas Claim 4 items may require students to organize information a bit more and contribute information to the problem in ways they have not been asked to do in past assessments. The wide range of acceptable responses for many Claim 4 items makes a handscoring requirement likely.
Adaptive Versions Lower difficulty by changing the given information (e.g., instead of volume, give radius of Earth).		
Source: Smarter Balanced Assessment Consortium Supplemental Samples		

Claim 1: Concepts and Procedures	**Grades:** HS	**Comparison to Traditional State Test Item(s)**
Item Stem For items 1a – 1e, determine whether each equation is True or False. 1a. $\sqrt{32} = 2^{\frac{5}{2}}$ True False 1b. $16^{\frac{3}{4}} = 8^2$ True False 1c. $4^{\frac{1}{2}} = \sqrt[4]{64}$ True False 1d. $2^8 = \left(\sqrt[3]{16}\right)^6$ True False 1e. $\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$ True False		The multi-part selected response items provide information on fluency and depth of understanding. Since these items ask students to respond to multiple parts, efficiency with certain skills is critical to students' success. These items measure depth of understanding as different parts of the item tap into slightly different mental processes. The consortium will explore the depth of understanding assumption in its cognitive labs during summer 2012.
Responses to this item will receive 0-2 points, based on the following: **2 points:** TTFTF **1 point:** TTFTT, TTFFF, TTTTF, TTFFT, TTTFF **0 points:** All other possibilities.		Multi-part selected response items can be scored as 0-1 or 0-2 depending on the depth of understanding required for the item (e.g., whether it is possible across the parts of the item to determine if a student has no understanding, some understanding or full understanding.
Source: Smarter Balanced Assessment Consortium Item Specifications MAT.HS.SR.1.00NRN.A.152		

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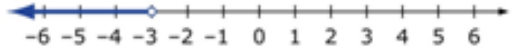
Claim 1: Concepts and Procedures | **Grades:** 8 – HS | **Comparison to Traditional State Test Item(s)**

Item Stem
Match each inequality in items 1 – 3 with the number line in items A – F that represent the solution to the inequality.

1. $-4x < -12$
2. $2(x + 2) < 8$
3. $5 - 2x < 2 - x$

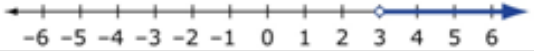
A


B


C


D


E


F


Content demands are typically higher. Compare to this traditional math items for Grade 8.

2 Write the inequality below in words.

$a \leq 2$

Graph the inequality on the number line. Label the intervals.

Source: Smarter Balanced Assessment Consortium Item Specifications
MAT.HS.TE.1.0AREI.I.088

Source: WA Released
http://www.k12.wa.us/Mathematics/pubdocs/QuickGuide_GR8.pdf

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Claim 2: Problem Solving	**Grades:** 4 – 6	**Comparison to Traditional State Test Item(s)**

Item Stem

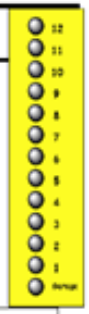
Elevators

Anna and Bob drive into the basement garage of an apartment block to visit their mother who lives on the 11th floor.

They decide to take different elevators.

They both enter the elevators at the same time.

This is what happens to the elevators.



Anna's elevator	Bob's elevator
The elevator waits 5 seconds before it starts moving.	The elevator waits 15 seconds before it starts moving.
It takes 7 seconds to go up each floor.	It takes 5 seconds to go up each floor.
The elevator does not stop before it reaches floor 11.	The elevator stops at floor 8 for 10 seconds.

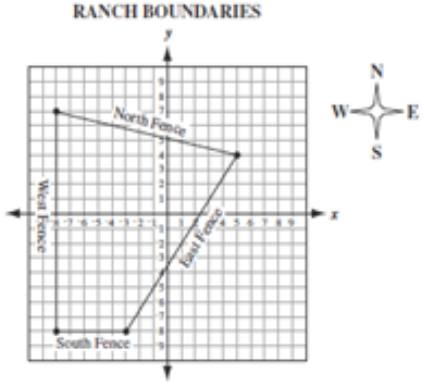
How long does it take Anna's elevator to reach floor 11? Show all the steps you take to solve this problem.

Who arrives at floor 11 first, Anna or Bob? Show all the steps you take to solve this problem.

Item Source: Smarter Balanced Assessment Consortium Mathematics Content Specifications

Contexts in new assessments are used to enhance content. Compare the example on the left (where the context is necessary) to this traditional assessment problem where the context adds unnecessary text.

5 The boundaries of a section of Anita's ranch are plotted on the coordinate plane below. Each vertex on the grid has integer coordinates.



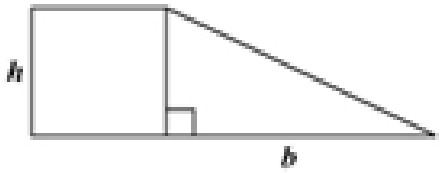
What is the slope of the segment that represents the east fence on the graph?

Item Source: 2011 FCAT Sample Questions
http://fcat.fldoe.org/pdf/sample/1011/math/FLS22267_Gr10_Mth_TB_WT_r3g.pdf

Claim 2: Problem Solving	**Grade:** HS

Item Stem

The figure below is made up of a square with height, h units, and a right triangle with height, h units, and base length, b units.



The area of this figure is 80 square units.

Write an equation for the height, h, in terms of b. Show all work necessary to justify your answer.

h = _____

Source: Smarter Balanced Assessment Consortium Item Specifications MAT.HS.CR.2.0ASSE.A.005

Claim 1: Concepts and Procedures	**Grades:** 7 – HS

Item Stem

A result of global warming is that the ice of some glaciers is melting. Twelve years after the ice disappears, tiny plants, called lichen, start to grow on the rocks.

Each lichen grows approximately in the shape of a circle. The relationship between the diameter of this circle and the age of the lichen can be approximated with the formula:

$d = 7.0 \times (t - 12)$ for $t \geq 12$

where d represents the diameter of the lichen in millimetres, and t represents the number of years after the ice has disappeared.

Using the formula, calculate the diameter of the lichen, 16 years after the ice disappeared. Show your calculation.

Adaptive Versions (Some adaptations may result in an appropriate item for Claim 2, 3, or 4)

Give a value for the diameter instead of time; include a measurement conversion; Graph the change in radius over time and discuss the shape of the graph; Examine the area of each lichen.

Source: PISA 2006 Released Item (M047: Lichen)

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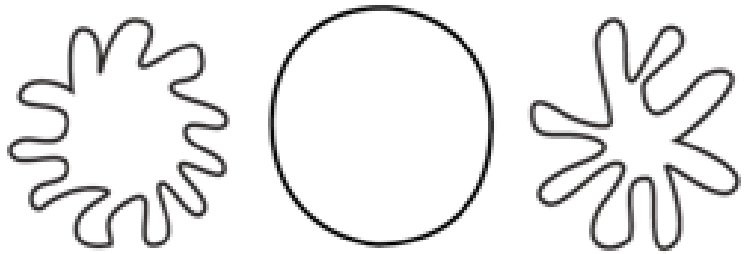
Sample Mathematics Items and Tasks
Middle / High School

Claim 3: Communicating Reasoning	**Grades:** 5 – 7

Item Stem



Which of the figures has the largest area? Explain your reasoning.

Source: PISA 2006 Released Item (M158: Shapes)

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Claim 2: Problem Solving	**Grades:** 7 – 8

Item Stem
In a sale, all the prices are reduced by 25%.

1. Julie sees a jacket that cost $32 before the sale. How much does it cost in the sale? Show your calculations.

In the second week of the sale, the prices are reduced by 25% of the previous week's price. In the third week of the sale, the prices are again reduced by 25% of the previous week's price. In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%. Explain why Julie is wrong.

3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay? Show your calculations.

4. Julie buys her jacket after the four reductions.
What percentage of the original price does she save? Show your calculations.

Source: Smarter Balanced Assessment Consortium Mathematics Content Specifications

Claim 4: Modeling and Data Analysis	**Grades:** 6 – 8

Item Stem

Pam is thirteen today.
She is holding a party at which she plans to play the game 'Wrap the mummy'.
In this game, players try to completely cover themselves with toilet paper.

A roll of toilet paper contains 100 feet of paper, 4 inches wide.

Will one toilet roll be enough to wrap a person?

Describe your reasoning as fully as possible.
(You will need to estimate the average size of a thirteen-year-old.)

Source: Smarter Balanced Assessment Consortium Mathematics Content Specifications

Claim 3: Communicating Reasoning	**Grades:** 6 – 8
Item Stem Max bought 2 items in a sale. One item was 10% off. One item was 20% off. Max says he saved 15% altogether. Is he right? Explain.	
Source: Smarter Balanced Assessment Consortium Mathematics Content Specifications	

Discussion and Questions